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Final Status Report

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A STUDY of DIGITAL TECHNIQUES for SIGNAL PROCESSING

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Summary of Prior Work on Grant

Prior reports covering this second year of the grant have summarized work in the following areas:

1. Adaptive Signal Processing:

   a. Adaptive Equalizers for digital transmission

   b. Adaptive array processing

2. Optimal adaptive control for data compression systems

3. Effects of quantization and roundoff noise in digital filters

4. Computer simulation of low error rate communication systems

5. Signal zero-crossings as information carriers in communication systems

6. Optimum and adaptive DPCM

Work in areas 3, 4, 5, 6 is essentially completed and so is not referred to in the next section summarizing ongoing work. The work on computer simulation (area 4) was summarized in an invited paper presented at the National Electronics Conference, Chicago, December 8, 1969, and appears in the published Proceedings of that conference. Copies of the paper are appended to this report. A detailed technical report covering the work of area 5 has been prepared and copies are also appended to this report. (A preliminary report covering early work was presented at the IEEE International Symposium on Information Theory, Ellenville, N.Y., January 1969.) A report covering area 6 in detail has been written and copies should be available shortly. Papers based on areas 5 and 6 are also being prepared for submission to the appropriate technical journals.

The first two areas represent continuing work. The status of work in area 1 is summarized below. Appended to this report is a paper submitted for publication that details some of the work carried out in this area.

Ongoing work in area 2, optimal adaptive control for data compression systems, is not reported on in detail here, since the effort during this reporting period has been primarily devoted to preparing programs to handle input Markov data. This work has taken somewhat longer than originally expected but most of the necessary writing and debugging of programs is now completed. Two tapes of raw data taken from one of the experiments conducted under the U. S. space program have been received from the NASA Goddard Space Flight Center and these will be used as input data to test out the optimal adaptive controllers developed, once all the programming work is completed.

In the section following we report on recent work in three areas:

1. Adaptive Signal Processing
2. Recursive Equalizers
3. Markov Processes in Nonlinear Detection and Estimation

Areas 2 and 3 represent new activity initiated during the period of this report. As noted in the section following they are both outgrowths of prior work under this grant. The work on recursive equalizers is based on ongoing work in adaptive equalizers, while that on Markov processes is based on work on recursive detection carried out during the first year of the grant.

Summary of Work During Reporting Period

1. Adaptive Signal Processing

a. Adaptive Equalizers for Digital Transmission

In previous reports (April 30, 1969, January 31, 1969, July 31, 1969) we outlined a new method for the adaptive equalization of digital signals transmitted through dispersive channels. The equalizer investigated was a non-recursive digital filter with coefficients adjusted iteratively. The new method uses a variable step size gradient search procedure rather than the fixed step size technique usually used, and the step sizes are chosen to provide the smallest possible distortion at the end of a specified number M of iterations. Analysis and computer simulation both verified that this technique resulted in a considerable improvement in convergence rate.

Two types of algorithms were described: a first order one in which the distortion, although minimized at the end of M intervals, may increase at intermediate intervals, and a second order one in which the distortion decreases monotonically at each iteration. This latter scheme is particularly appropriate for an equalizer in a tracking mode.

We have now completed the investigation of the second order scheme, considering its performance in noise, verifying its operation through computer simulation, and analyzing the performance of sub-optimum second order schemes using constant coefficient algorithms. A paper reporting the results of the second order scheme has been submitted for presentation at the forthcoming International Symposium on Information Theory, June 1970, in Holland. It has also been submitted for publication in the IEEE Transactions on Information Theory. Three copies of this paper are enclosed with this report.

We have also been invited to present a paper on the first order scheme at the forthcoming Princeton Conference on Information Sciences and Systems, March 1970. The full paper is currently being written up and will also be submitted for publication in the Information Theory Transactions.

Details of the second order scheme appear in the accompanying paper noted above. In summary it suffices to say that an investigation of its performance in noise indicates that the asymptotic variance of the algorithm has a slightly larger upper bound than that of the present first order fixed step algorithm. However, a computer simulation for an input signal to noise ratio of 30 db shows that for large intersymbol interference the improvement in the convergence of the mean more than compensates for the small increase in variance. For moderate intersymbol interference the simulation showed no variance increase.

As noted in prior status reports the variable coefficients of the optimum second order algorithm were found to converge rapidly (after only a few iterations) to asymptotic constant values. It was therefore conjectured that suboptimum second algorithms with constant coefficients throughout might profitably be used. The detailed analysis of these optimum schemes also appears in the accompanying paper. The results indicate that convergence of this algorithm is assured for a wide range of coefficients, and that there exists a region of improvement in convergence over the fixed step size, first order algorithm. Specifically, the suboptimum constant coefficient second order algorithm is given by:

$$\underline{C}^{k+1} = \underline{C}^{k} - \alpha \left(\frac{1}{2} \nabla_{\underline{C}^{k}} \epsilon\right) + \beta \left(\underline{C}^{k} - \underline{C}^{k-1}\right),$$

with $\underline{C}$ the vector of the digital filter coefficients that are to be obtained by iteration, and α and β the constant coefficients. Letting $\beta = 0$

converts this into the usual fixed step size gradient search procedure.

The analysis indicates that convergence is assured if

$$\alpha < \frac{2(1+\beta)}{\lambda_\mu}$$

$$\beta < 1$$

with λ_μ the upper bound to the eigenvalues of the correlation matrix of the incoming signal data.

An optimum pair of values for α and β are found to exist in the sense of maximum convergence rate. This pair of values are exactly the asymptotic values of the variable coefficient optimum scheme. For $\beta < 2\beta_{opt}$ this suboptimum constant coefficient second order algorithm always converges more rapidly than the fixed step size first order scheme ($\beta = 0$).

b. Adaptive Array Processing

Initial work on this problem was described in the Status Report of July 31, 1969. The problem is to automatically make an array of isotropic detectors form a beam in a desired direction in space when unknown interfering noise is present so as to maximize the output signal-to-noise ratio (SNR). Iterative gradient techniques are used to do this.

One question that immediately arises is what approach do we use; i.e. we can view the detectors either as an antenna array, and, by using the concept of an antenna pattern, solve for those current and phase excitations which produce the maximum SNR, or, alternatively, we can view the detector array as a multichannel filter, and, using statistical communication theory, solve for those filter coefficients which maximize the SNR. However, since there is only one physical problem, these two different approaches must ultimately yield equivalent results. This has been demonstrated. Moreover, when the noise is monochromatic, it turns out that an analogy can be drawn between intermediate terms in the two approaches. Thus, this phase of the research was concerned with demonstrating the equivalence between the "antenna pattern" and "multichannel filter" points of view in designing optimum arrays. Specifically we investigated:

1. Optimum array design using the "antenna pattern" point of view, assuming the incident noise power is known.

2. Optimum array design using the "multichannel filter" point of view, assuming the noise space-time correlation function is known.

3. The general relationship between the space-time correlation function and the incident noise power.

4. The equivalence of parts 1 and 2 above under a monochromatic noise assumption.

Next, using the "antenna pattern" point of view we investigated the sensitivity of the SNR to random excitation errors and random errors in the detector locations. This sensitivity is essentially given by the super-gain ratio, and through the use of the analogy described above, we were able to find an analog to the super-gain ratio in terms of communication theory quantities (e.g. correlation functions). It was noted that when we use arrays of detectors separated by one-half wavelength or less, this sensitivity factor became very large when the optimum currents and phases of part 1 above were used, thus indicating that we should not try to design our antenna pattern or multichannel filter coefficients on the basis of maximizing the SNR alone, but rather on the basis of maximizing the SNR subject to a contraint on the supergain ratio as done by Lo, Lee, and Lee (Proceedings of IEEE, Vol. 54, No. 8, August 1966). Numerical determination of the optimum excitations to use when we constrain the supergain ratio is now being investigated.

Next, we tried to analytically consider adaptive algorithms which would maximize the SNR subject to a constraint on the supergain ratio when unknown interfering noise is present. Because the SNR and supergain ratio are nonlinear quantities, it turned out to be exceedingly difficult to prove convergence of the algorithms to the optimum solution, or to find the algorithms' rates of convergence. Thus, solely for the purpose of mathematical tractability (the nonlinear problem will be simulated on a computer to obtain some numerical indication of convergence and convergence rates), we considered adaptive algorithms which minimize the mean square error (MSE) subject to a linear constraint. Specifically, we found the Lagrange solution to the problem of minimizing the MSE subject to a linear constraint and then proved that an algorithm of the form

$$\underline{W}_{j+1} = \underline{W}_j - kP\nabla_{\underline{W}_j}(MSE)$$

converges to the Lagrange solution in real time, with an easily expressible bound on the convergence rate. Here k is the step size constant, P is a matrix projection operator (see J. B. Rosen, Journal of SIAM, vol. 8, March 1960 and vol. 9, December 1961), and $\nabla_{\underline{W}_j}$ is the gradient of the MSE with respect to $\underline{W}_j$. We also proved convergence and found bounds on the rate of convergence when the gradient was (1) known exactly (2) estimated, and (3) estimated by a noisy estimate.

We next plan to simulate these algorithms on a computer and compare the simulated and theoretical results.

2. Recursive Equalizers

All adaptive equalizers thus far investigated utilize non-recursive digital filters as processors. That is, they consider filters of the type

$$y_n = \sum_{\ell=1}^{k} a_\ell x_{n-\ell}$$

where y_n is the filter output at time $t = nT$ due to the linear combination of k previous inputs $x_{n-\ell}$. The filter coefficients a_ℓ are found using various iterative search techniques.

Most current work on digital filters as applied to more classical filtering problems (rather than adaptive equalization) assumes a recursive structure. In this case we have

$$y_n = \sum_{j=1}^{m} b_j y_{n-j} + \sum_{\ell=1}^{k} a_\ell x_{n-\ell} \, .$$

Fewer coefficients (or a simpler structure) may be needed for such a filter as contrasted with the non-recursive type. The possible use of such structures for adaptive equalization is tempting because of the possibility of improved computational simplicity.

It is relatively easy to iteratively find the coefficients a_ℓ of the non-recursive filter since the error is a linear function of the coefficients. For the non-recursive structure, however, a non-linear relationship exists between the error and the coefficients a_ℓ and b_j. It is then difficult to develop an iterative scheme to search for the optimum coefficients. We have developed an alternate approach. A pseudo-error is created which preserves the linearity between this new error and the coefficients. Iterative schemes for searching for "optimum" coefficients to minimize this pseudo-error can then easily be found.

Several problems exist and are currently being investigated. Among these is the relationship between the pseudo-error and the actual error. We have found that in many cases the coefficients that minimize the pseudo-error also yield small actual errors. However in some cases the filter produced was unstable and whereas the pseudo-error was small, the actual error was large. The algorithms are now being modified to prevent this.

The non-recursive equalizer is incapable, in general, of completely equalizing the channel. But if the channel spread is finite a recursive equalizer can be found to yield zero error - it need only have a sufficient (but finite) number of coefficients. One difficulty is that our algorithm breaks down when this is possible. Several modifications are being considered to alleviate this problem.

We have also found that in some cases the recursive filter performs better than a non-recursive filter with an equivalent number of coefficients. But in other cases the reverse is observed. Investigation is continuing into this phenomenon.

3. Markov Processes in Nonlinear Detection and Estimation

In previous reports on this grant covering the period February 1, 1968 - January 31, 1969, we have described the application of recursive techniques to digital signal processing. The specific problem investigated was that of signal detection in noise. The recursive structure of the digital processor obtained depended on the vector Markov property of the process being sampled.

We report here on work recently begun extending these results and those of other investigators to more general problems in nonlinear detection and estimation involving Markov processes.

The estimation of a random signal transmitted through a noisy channel and the detection of the presence or absence of signals under similar conditions are two well known and important problems in Communication Theory.

If x_t is a random signal and y_t is a nonlinearly correlated noisy observation, then the optimum minimum mean square sense (MMS) estimate $\hat{x}_t$ of the signal x_t given the past observations $\{y_s, t_o \leq s \leq t\}$ is the conditional mean of x_t given these past observations of y_t. Letting H_0 and H_1 be the two hypotheses of the binary detection problem (H_1 signal plus noise, H_0 noise alone) the optimum (minimum probability of error) detector computes the likelihood ratio for the given observation $\{y_s, t_o \leq s \leq t\}$ and compares it with a threshold.

In recent years the modelling of x_t and y_t as a vector diffusion process defined as the solution of an Ito vector stochastic differential equation

$$dx_t = m(x_t, y_t, t)dt + \sigma(x_t, t)db^x_t$$

$$dy_t = M(x_t, y_t, t)dt + \Sigma(y_t, t)db^y_t$$

has found wide interest [1], [2], [3] mainly because of the following reasons.

(1) The successful solution of the particular case of linear equations of the form

$$dx_t = m(t)x_t dt + \sigma(t)db^x_t$$

$$dy_t = M(t)X_t dt + \Sigma(t)db^y_t$$

has led to the well known Kalman-Bucef filter ([4]) and the optimum detector can be implemented as an estimator-correlator [5].

(2) The Markovian assumption reflects itself in the "naturality" with which causal structures are obtained for the optimum filter and the optimum detector. If digital processing is used, the recursive nature of the solution greatly reduces the memory requirements for the processor [6].

(3) A very extensive mathematical literature is available on Markov processes and these are "easily" described in terms of transition probability functions which under suitable conditions satisfy well known partial differential equations of the parabolic type (Kolmogorov's equations).

The extension of the above results to the much more difficult case of nonlinear dynamics has been done by several authors, ([7], [8], [9]). Unfortunately the results obtained are difficult to evaluate or implement since they are given in terms of stochastic equations.

Meaningful approximations to such results are needed to solve problems of practical interest. Approximate expressions for both the conditional mean and the likehood ratio have been obtained in terms of the solution of a "classical" (i.e., non stochastic) parabolic Partial Differential Equation (PDE). This PDE takes into account both the signal and the observation dynamics as well as the actual observation being performed. If the observation noise increases without bound, the PDE goes into the forward Kolmogorov equation for the transition density and the conditional mean goes into the a priori mean, as required. As a check, the Kalman-Bucy equations for the conditional mean are obtained if a linear model for the signal and the observation is assumed. Work is under way to find explicit solutions of the PDE for the limit cases of low signal-to-noise ratio and high signal-to-noise ratio.

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